Optimum Equipment Maintenance/Replacement Policy Part 1: Dynamic Programming Approach

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This is the first part of a two-part article on the optimization of equipment maintenance and/or replacement policy. Over a given life-span of equipment, optimum policy is determined based on present costs, inflation rates, operating characteristics, future equipment developments, and other factors. A computer program utilizing the dynamic programming technique together with a numerical example is included.

Part II will present an optimum policy determined by a stochastic model such as the Markovian decision processes. In such a model, a long-term expected return/cost is optimized including the probability of equipment breakdown as functions of some performance criteria.

I. Introduction

One of the basic tasks of operating and maintenance engineering is the replacement of old machinery and obsolete tools by new and modern ones. As equipment deteriorates with use, or degrades relative to the performance of newer models or improvements in design, there comes a time for operation and maintenance engineers to make decisions regarding replacement or maintenance. When the outlay for new equipment is made, the financial loss due to the stoppage of operation and the cost of teaching new skills should be compensated for in the trade-off studies by an increase in productivity and a decrease in maintenance and operating costs.

The optimal repair and/or replacement policies under various assumptions concerning present costs, discount rates, operating characteristics, and future equipment developments are essential to the successful operation of a facility. Since decisions of this type must be made periodically, depending upon the fundamental time period and the type of process in consideration, it is clear that this maintenance policy is a multistage decision process of the dynamic programming type.

Dynamic programming (or dynamic optimization) is a mathematical technique often used for making a sequence of interrelated decisions (Refs. 1 and 2), provides a systematic procedure for determining the combination of decisions that optimizes the overall effectiveness measure. It is a general approach to many dynamic problems, but the particular equations used must be developed to fit each individual problem.

II. Computational Algorithm

The optimal equipment-replacement policies and the decision process using dynamic programming may be described as follows:

It is assumed that a machine or a piece of equipment requires a certain initial capital, generates a certain revenue, requires a certain amount of care, and can be replaced by a new machine at any time. The revenue, the maintenance cost, and the rebate on trade-in are all taken to depend on the age of the machine in known fashions. With this information, a best set of decisions, such as to KEEP, to OVERHAUL, or to REPLACE, need to be determined in order to optimize the total profit/cost over a given number of years. Thus, the objective is to optimize the total profit/cost, Y,

$$Y = \sum_{j=1}^{n} F_j(i) \quad \text{for } j = 1, 2, \cdots, n \tag{1}$$

with respect to decisions, D_j, where D_j can be "Keep", "Overhaul", or "Replace" for $j = 1, 2, \cdots, n$ years. The function $F_j(i)$ is defined as the value at year j of the overall profit/cost from a machine that is (i) years old, where an optimal policy is employed for the remainder of process operation left.

It is assumed that the process lasts n years, and then stops. Hence,

$$F_{n+1}(i) = 0 \tag{2}$$

By the principle of optimality, the overall profit/cost of the j^{th} time period with a given decision can be obtained by

$$F_j^*(i) = OPT\ \{F_j^K(i), F_j^0(i), F_j^R(i)\} \tag{3}$$

Optimum function $F_j^*(i)$ can be determined by:

$$\text{Keep:}\ F_j^K(i) = R_j(i+1) - U_j(i+1) + F_{j-1}(i+1) \tag{4}$$

$$\text{Overhaul:}\ F_j(i) = R_j(k+1) - U_j(k+1) - O_j(i+1) + F_{j-1}(k+1) \tag{5}$$

$$\text{Replace:}\ F_j^R(i) = R_1(1) - U_1(1) - C_j(i+1) + F_{j-1}^*(1) \tag{6}$$

where (i) is the age of the equipment at the end of the previous operation period, (k) is the equivalent age of the equipment after the overhaul, and (j) is the current operation period. The revenue R, operating/maintenance U, overhaul O, and replacement C, cost information may be tabulated or represented by analytical expressions.

III. Numerical Example

Suppose a piece of equipment (t) years old with no salvage or resale value has the annual income R in thousands of dollars; R, after all running expenses have been met, is given by

$$R(t) = \begin{cases} 25 - t^2 & \text{for } 0 < t \leqslant 4 \\ 0 & \text{for } t > 4 \end{cases} \tag{7}$$

Let the cost of replacement C be \$21,000, and the equipment life be 5 operating years; then the optimal future equipment replacement policy must be determined if the present equipment is 2-years old and if a decision must be made annually on the basis of a maximum total 5-year profit.

The annual profit $P_j(i)$ will be, then, the difference between income and costs

$$P_j(i) = \begin{cases} 25 - i^2 & \text{for } i \leqslant 4 \\ 0 & \text{for } i > 4 \end{cases} \tag{8}$$

if we keep the unit. If we replace it, the age of the equipment is $i = 0$, and

$$P_j(0) = 25 - (0)^2 - 21 = 4 \tag{9}$$

where the additional charge is the cost of replacement. Since the unit has no resale value, the net profit from a new unit is independent of the age of the unit being replaced, and we may write $P_j(0) = 4$ as the first-year profit.

We must now find the best set of decisions, KEEP (K), OVERHAUL (O), or REPLACE (R) to maximize the profit from the 5-year process life. Thus we wish to maximize

$$Y = \sum_{j=1}^{5} P_j(i) \tag{10}$$

with respect to D_j, where D_j can take the value K_j or R_j for $j = 1, 2, 3, 4, 5$.

By a direct approach we could write all possible decision considerations and find the optimum policy. Thus, as in Table 1, one arbitrary combination *K-R-K-K-R* gives a total profit of \$74,000.

This direct approach requires 2^n evaluations, or $(2)^5 = 32$ for the example, all of which need to be compared with each other to determine the largest for the 5-year period.

Since the problem is a multistage process, it can be carried out more simply by dynamic programming with a recursive function:

$$F_j^*(i) = OPT_{K_j, R_j} \{P_j(i+1) + F_{j-1}^*(i+1)\} \qquad (11)$$

For example, for a 3-year old item with 2 years to go, we will have the form

$$F_2(3) = OPT \begin{cases} P_2(3) + f_1^*(4) & \text{if keep} \\ P_2(0) + f_1^*(1) & \text{if replace} \end{cases} \qquad (12)$$

The numerical value of the two possibilities are:

$$F_2(3) = \begin{cases} 16 + 9 = 25 & \text{if keep} \\ 4 + 24 = 28 & \text{if replace} \end{cases} \qquad (13)$$

It is therefore more profitable to replace this equipment in this situation.

The complete solution using dynamic programming is given in Appendix A. For the given numerical example, there are two alternate policies: One policy is *K-K-R-K-K*; the other is *K-R-K-K-K*. Both policies result in a 5-year profit of $86,000. A comparison of the two policies show that more profit is returned in the first 2 years of the 5-year period with the first optimal policy than the second, and this might be preferred if the accuracy of the cost estimates might be questioned after several years of operation.

IV. Summary

A computer program utilizing the dynamic programming techniques for optimal equipment replacement policy is written in the BASIC language and is tested on a Hewlett-Packard 2647A graphics terminal. The computer program listing is given in Appendix B.

The computer program is capable of performing optimal repair and replacement policies over a given operating period under various assumptions concerning present costs, discount rates, operating characteristics, and future technology development. It provides a tool for management to decide replacement schedules and to know when and how much capital outlay will be required.

Another optimization technique including the probability of equipment break-down as a function of year, age, and other factors will be presented in Part II at a later date.

Acknowledgment

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References

1. Bellman, R. E., Dreyfus, S. E., *Applied Dynamic Programming*, Princeton University Press, Princeton, N.J., 1962.

2. Beveridge, G.S.G., Schechter, R. S., *Optimization: Theory and Practice*, McGraw-Hill Book Company, 1970.

Table 1. Direct solution of numerical example

Age at end of previous period	Policy	Age at start of period	Profit, \$K
2	*K*	2	21
3	*R*	0	4
1	*K*	1	24
2	*K*	2	21
3	*R*	0	4
Total			74

Appendix A

Numerical Example Printout

```
* * * * * * * * *  INPUT INFORMATION  * * * * * * * * *


                   OUTPUT OPTION                        0
                   MACHINE STARTS AT AGE                2
                   AGE RETURN-BACK AFTER OVERHAUL       2
                   NUMBER OF TIME PERIODS               5
                   DISCOUNT PERCENTAGE                  0
                   MAXIMIZATION FLAG                    1
                   TABULATED COST/RETURN INPUT FLAG     0


* * * * * * * * * * * * * * * * * * * * * * * * * * * * *
```

```
-------------------------------- POLICY 1 ----------------------------------
PERIOD      EQUIP AGE        DECISION          VALUE
------      ---------        --------          -----
 1           2               KEEP               21
 2           3               KEEP               16
 3           4               REPLACE            4
 4           1               KEEP               24
 5           2               KEEP               21
----------------------------------------------------------------------------

TOTAL COST/PROFIT OVER 5 PERIODS                86
```

```
-------------------------------- POLICY 2 ----------------------------------
PERIOD      EQUIP AGE        DECISION          VALUE
------      ---------        --------          -----
 1           2               KEEP               21
 2           3               REPLACE            4
 3           1               KEEP               24
 4           2               KEEP               21
 5           3               KEEP               16
----------------------------------------------------------------------------

TOTAL COST/PROFIT OVER 5 PERIODS                86
```

Appendix B

Computer Program Listing

```
10 REM..............................................................REM
15 REM..........                                            ..........REM
20 REM..........          EQUIPMENT REPLACEMENT              ..........REM
25 REM..........      BY DYNAMIC PROGRAMING APPROACH         ..........REM
30 REM..........                                            ..........REM
35 REM..........      T. CHARNG        1 OCTOBER 1981        ..........REM
40 REM..........                                            ..........REM
45 REM..............................................................REM
50 DIM R(10,10),U(10,10),C(10,10),O(10,10),D(10,10,3),Opt(10,10)
55 DIM Tie(10,10),Iloc(10),P(10),F(3)
60 REM
65 INTEGER Age,Period,D,Tie,Back,Alter,P
70 REM...................ASSIGN PRINTOUT FILE "OUTPUT"..............REM
75 Ko=6
80 ASSIGN "OUTPUT" TO #Ko
85 REM
100 REM.............................................................REM
105 REM...................INPUT SECTION.............................REM
110 READ Iprint           !  PRINTOUT OPTION
115 READ Age              !  AGE OF THE MACHINE TO BEGIN WITH
120 READ Nu               !  AGE OF THE MACHINE AFTER OVERHAUL
125 READ Period           !  NUMBER OF THE TIME PERIODS
130 READ Disc             !  DISCOUNT PERCENTAGE
135 READ Maxi             !  MAXIMIZATION FLAG
140 READ Itable           !  TABULATED COST/RETURN INPUTS
145 REM...................PRINTOUT GENERAL INFORMATION..............REM
150 IF Iprint>=0 THEN GOSUB 5400
160 REM...................TABULATED COST/RETURN.....................REM
165 IF Itable>0 THEN GOSUB 1000
170 REM.............................................................REM
175 PRINT #Ko;LIN(2)
180 PRINT #Ko;"* * * * * * * * * * * * * * * * * * * * * * * * * * * * *"
185 REM.............................................................REM
190 IF Itable<=0 THEN GOSUB 4000
195 REM.............................................................REM
200 Disc=1/(1+Disc/100)
203 Ierr=0
205 REM.............................................................REM
210 FOR J=1 TO Period
215 Ilast=Age+Period-J
220 FOR I=1 TO Ilast
225 Tie(I,J)=0
230 IF J<>1 THEN 260
235 F(1)=R(I+1,1)-U(I+1,1)
240 F(2)=R(Nu+1,1)-U(Nu+1,1)-O(I+1,1)
245 F(3)=R(1,1)-U(1,1)-C(I+1,1)
250 GOTO 280
255 REM
260 F(1)=R(I+1,J)-U(I+1,J)+Opt(I+1,J-1)*Disc
270 F(2)=R(Nu+1,J)-U(Nu+1,J)-O(I+1,J)+Opt(Nu+1,J-1)*Disc
275 F(3)=R(1,1)-U(1,1)-C(I+1,J)+Opt(1,J-1)*Disc
280 IF Iprint>1 THEN GOSUB 5100
```

```
290 REM.....................................................REM
295 Opt(I,J)=F(1)
300 D(I,J,1)=1
305 FOR K=2 TO 3
310 IF Maxi>0 THEN 350
315 IF Opt(I,J)<=F(K) THEN 360
320 Opt(I,J)=F(K)
325 FOR L=1 TO K-1
330 D(I,J,L)=0
335 NEXT L
340 D(I,J,K)=K
345 GOTO 375
350 IF Opt(I,J)>=F(K) THEN 360
355 GOTO 320
360 IF Opt(I,J)<>F(K) THEN 375
365 D(I,J,K)=K
370 Tie(I,J)=Tie(I,J)+1
375 NEXT K
380 NEXT I
385 PRINT #Ko
390 NEXT J
400 REM.....................................................REM
410 Alter=1
415 IF Iprint>1 THEN GOSUB 5020
420 Back=0
425 I1=0
430 I=Age
435 J=Period
440 REM
445 I1=I1+1
450 Iloc(I1)=I
455 V(J)=Opt(I,J)
460 FOR K=1 TO 3
465 IF D(I,J,K)<=0 THEN 535
470 P(J)=D(I,J,K)
475 IF D(I,J,K)<>1 THEN 495
480 IF Iprint>1 THEN PRINT #Ko;TAB(20);J,I,"KEEP",Opt(I,J)
485 I=I+1
490 GOTO 560
495 IF D(I,J,K)<>2 THEN 515
500 IF Iprint>1 THEN PRINT #Ko;TAB(20);J,I,"OVERHAUL",Opt(I,J)
505 I=Nu
510 GOTO 560
515 IF D(I,J,K)<>3 THEN 540
520 IF Iprint>1 THEN PRINT #Ko;TAB(20);J,I,"REPLACE",Opt(I,J)
525 I=1
530 GOTO 560
535 NEXT K
540 IF Ierr>0 THEN 65000
541 Ierr=1
542 Iprint=2
545 PRINT #Ko;TAB(5);"$$$$$$$$$$  SOMETHING IS WRONG AT PERIOD";J;
546 PRINT #Ko;"AGE";I;"D(";I;",";J;",";K;")=";D(I,J,K)
547 PRINT #Ko;LIN(1);"EXAMINE THE RESULTS CAREFULLY"
548 PRINT #Ko
550 GOTO 220
```

```
560 REM.................................................................REM
565 Ii=Iloc(I1)
570 IF Tie(Ii,J)<=0 THEN 590
575 Back=1
580 Tie(Ii,J)=Tie(Ii,J)-1
585 D(Ii,J,K)=0
590 J=J-1
595 IF J>=1 THEN 440
600 REM.................................................................REM
605 GOSUB 5000
610 FOR J=Period TO 1 STEP -1
615 Jp=Period-J+1
620 Ii=Iloc(Jp)
625 Vp=V(J)
630 IF J>1 THEN Vp=Vp-V(J-1)
635 IF P(J)=1 THEN PRINT #Ko;TAB(20);Jp,Ii,"KEEP",Vp
640 IF P(J)=2 THEN PRINT #Ko;TAB(20);Jp,Ii,"OVERHAUL",Vp
645 IF P(J)=3 THEN PRINT #Ko;TAB(20);Jp,Ii,"REPLACE",Vp
647 NEXT J
655 PRINT #Ko;TAB(20);"------------------------------------------------------------"
660 PRINT #Ko;LIN(1);TAB(20);"TOTAL COST/PROFIT OVER";Period;"PERIODS";
665 PRINT #Ko;TAB(10);Opt(Age,Period)
670 PRINT #Ko
680 REM.................................................................REM
685 Alter=Alter+1
690 IF Back=1 THEN 415
695 GOTO 65000
1000 REM...................SUB FOR TABULATED COST/RETURN INPUTS.......REM
1010 FOR I=1 TO Period+2
1020 FOR J=1 TO Period
1030 READ R(I,J)          !  REVENUE
1040 NEXT J
1050 NEXT I
1060 REM
1070 FOR I=1 TO Period+2
1080 FOR J=1 TO Period
1090 READ U(I,J)          !  OPERATION/MAINTENCE
1100 NEXT J
1110 NEXT I
1120 REM
1130 FOR I=1 TO Period+2
1140 FOR J=1 TO Period
1150 READ C(I,J)          !  REPLACRMENT
1160 NEXT J
1170 NEXT I
1180 REM
1190 FOR I=1 TO Period+2
1200 FOR J=1 TO Period
1210 READ O(I,J)          !  OVERHAUL
1220 NEXT J
1230 NEXT I
1240 RETURN
```

```
2000 REM...................INPUT DATA BLOCK..........................REM
2010 DATA 0                ! PRINTOUT OPTION
2020 DATA 2                ! AGE OF THE MACHINE TO BEGIN WITH
2030 DATA 2                ! AGE OF THE MACHINE AFTER OVERHAUL
2040 DATA 5                ! NUMBER OF THE TIME PERIODS
2050 DATA 0                ! DISCOUNT PERCENTAGE
2060 DATA 1                ! MAXIMIZATION FLAG
2070 DATA 0                ! TABULATED COST/RETURN INPUTS
2100 REM...................REVENUE MATRIX R(I,J).....................REM
2110 DATA 155,150,140,135,125
2120 DATA 140,135,125,120,115
2130 DATA 125,110,110,115,100
2140 DATA 105,105,110, 90, 95
2150 DATA 100,105, 80, 90, 70
2160 DATA 100, 70, 80, 65, 70
2170 DATA  60, 70, 65, 70, 60
2200 REM...................MAINTENCE/OPERATION COST U(I,J)............REM
2210 DATA   5,   5, 5, 10, 10
2220 DATA  10, 10, 10, 10, 10
2230 DATA  10, 10, 10, 15, 20
2240 DATA  10, 15, 15, 20, 20
2250 DATA  15, 20, 25, 25, 25
2260 DATA  20, 25, 25, 30, 30
2270 DATA  30, 30, 30, 35, 45
2300 REM...................REPLACEMENT COST C(I,J)...................REM
2310 DATA 220,220,220,210,210
2320 DATA 225,230,220,220,215
2330 DATA 240,230,230,220,220
2340 DATA 240,240,225,225,250
2350 DATA 250,230,230,255,255
2360 DATA 235,235,260,260,260
2370 DATA 240,265,265,265,270
2400 REM...................OVERHAUL COST O(I,J)......................REM
2410 DATA 55, 55, 55, 60, 60
2420 DATA 60, 60, 60, 60, 60
2430 DATA 60, 60, 60, 65, 70
2440 DATA 60, 65, 65, 70, 70
2450 DATA 65, 70, 75, 75, 75
2460 DATA 60, 65, 65, 80, 80
2470 DATA 80, 80, 80, 85, 95
3000 REM...................END OF DATA BLOCK.........................REM
4000 REM.....................................................REM
4020 FOR J=1 TO Period
4030 R(1,J)=25
4040 C(1,J)=21
4050 U(1,J)=0
4060 O(1,J)=10
4070 FOR I=2 TO Period+Age
4080 Im1=I-1
4090 R(I,J)=R(1,J)-Im1*Im1
4100 IF Im1>4 THEN R(I,J)=0
4110 C(I,J)=C(1,J)
4120 O(I,J)=O(1,J)
4130 U(I,J)=U(1,J)
4140 NEXT I
4150 NEXT J
4160 RETURN
```

```
5000 REM.............................................................REM
5002 PRINT #Ko;LIN(2)
5004 PRINT #Ko;TAB(20);"-------------------- POLICY";Alter;"-------------------------"
5006 PRINT #Ko;TAB(20);"PERIOD","EQUIP AGE","DECISION","VALUE"
5008 PRINT #Ko;TAB(20);"------","---------","--------","-----"
5010 RETURN
5020 REM.............................................................REM
5022 PRINT #Ko;LIN(2)
5024 PRINT #Ko;TAB(20);"-------------------- POLICY";Alter;"-------------------------"
5026 PRINT #Ko;TAB(20);"PERIOD","EQUIP AGE","DECISION","CUMULATED VALUE"
5028 PRINT #Ko;TAB(20);"------","---------","--------","---------------"
5030 RETURN
5100 REM.............................................................REM
5120 PRINT #Ko;TAB(5);"PERIOD=";J,"EQUIP AGE=";I;
5130 PRINT #Ko;TAB(5);"KEEP=";R(I+1,J);"-";U(I+1,J);
5140 PRINT #Ko;"+";Opt(I+1,J-1);"*";Disc;"=";F(1)
5150 PRINT #Ko;TAB(33);"RENU=";R(Nu+1,J);"-";U(Nu+1,J);"-";O(I+1,J);
5160 PRINT #Ko;"+";Opt(I+1,J-1);"*";Disc;"=";F(2)
5170 PRINT #Ko;TAB(33);"REPL=";R(1,J);"-";U(1,J);"-";C(I+1,J);
5180 PRINT #Ko;"+";Opt(1,J-1);"*";Disc;"=";F(3)
5190 RETURN
5300 REM.............................................................REM
5320 PRINT #Ko;LIN(20);TAB(5);" J "," I ","D(I,J,K)","OPT(I,J)","TIE(I,J)"
5325 PRINT #Ko;TAB(5);"---","---","--------","--------","--------"
5330 PRINT #Ko
5335 FOR J=1 TO Period
5340 FOR I=1 TO Period+Age-J
5345 PRINT #Ko;TAB(5);J,I,D(I,J,1);D(I,J,2);D(I,J,3),Opt(I,J),Tie(I,J)
5350 NEXT I
5355 PRINT #Ko
5360 NEXT J
5365 RETURN
5400 REM.............................................................REM
5410 PRINT #Ko;LIN(2)
5415 PRINT #Ko;"* * * * * * * * *  INPUT INFORMATION  * * * * * * * * *"
5420 PRINT #Ko;LIN(2)
5425 PRINT #Ko;TAB(20);"OUTPUT OPTION                            ";Iprint
5430 PRINT #Ko;TAB(20);"MACHINE STARTS AT AGE                    ";Age
5435 PRINT #Ko;TAB(20);"AGE RETURN-BACK AFTER OVERHAUL           ";Nu
5440 PRINT #Ko;TAB(20);"NUMBER OF TIME PERIODS                   ";Period
5445 PRINT #Ko;TAB(20);"DISCOUNT PERCENTAGE                      ";Disc
5450 PRINT #Ko;TAB(20);"MAXIMIZATION FLAG                        ";Maxi
5455 PRINT #Ko;TAB(20);"TABULATED COST/RETURN INPUT FLAG         ";Itable
5460 RETURN
5500 REM.............................................................REM
65000 REM
65010 PRINT #Ko;LIN(5)
65020 PRINT #Ko;TAB(25);"* * * * * * * * * * * * * * * * * * * * * * *"
65030 PRINT #Ko;TAB(25);"* *                                       * *"
65040 PRINT #Ko;TAB(25);"* *              END OF TASK              * *"
65050 PRINT #Ko;TAB(25);"* *                                       * *"
65060 PRINT #Ko;TAB(25);"* * * * * * * * * * * * * * * * * * * * * * *"
65070 REM
65080 COMMAND "M F H HP-IB#1"
65090 REM
65100 END
```